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Higgs plus jet production in bottom quark annihilation at next-to-leading order

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Abstract

The cross section for Higgs+jet production in bottom quark annihilation is calculated through NLO QCD. The five-flavour scheme is used to derive this contribution to the Higgs+jet production cross section which becomes numerically important in the MSSM for large values of $\tan\beta$. We present numerical results for a proton collider with 14 TeV center-of-mass energy. The NLO matrix elements for $d\sigma/dp_T$ are then combined with the total inclusive cross section in order to derive the integrated cross section with a maximum cut on p_T at next-to-next-to-leading order.

1 Introduction

The Higgs mechanism [1–3] plays a central role in both the Standard Model (SM) [4–6] and its supersymmetric extensions [7]. The gauge bosons and quarks acquire masses through interactions with Higgs fields. Up to now, the search for the physical Higgs boson has been unsuccessful but has led to the exclusion of a certain Higgs mass range [8, 9]. In combination with the fits of electro-weak precision data to higher order perturbative calculations this leads to a rather small range of allowed values for a SM Higgs boson mass [10].

Supersymmetric (SUSY) theories require an enlarged Higgs sector. The minimal SUSY extension of the SM leads to five physical Higgs bosons. Due to the larger number of free parameters in SUSY, the sensitivity of experimental data to Higgs bosons is weaker than in the SM [11, 12].

The Large Hadron Collider (LHC) is expected to find a Higgs boson if it exists. To do this, various production and decay channels must be considered. The relative utility of each channel depends strongly on the Higgs mass and couplings. While in the SM gluon fusion is the Higgs production process with the largest cross section by far, in SUSY theories with large $\tan\beta$, Higgs production in association with bottom quarks is dominant (for reviews, see Refs. [13, 14]; detailed studies of the relative importance of gluon fusion and bottom annihilation have been performed in Ref. [15–17]). This is because in this region of the SUSY parameter space the $Hb\bar{b}$ coupling is enhanced relative to the SM.

Assuming that only four quark flavours and the gluon make up the proton (the so-called “four-flavour scheme” or 4FS), the dominant leading order Feynman diagram for this



Figure 1: Leading order diagrams for associated $b\bar{b}h$ production in the (a) four and (b) five flavour scheme.

process is shown in Fig. 1(a). If the bottom quark was massless, integration over phase space would lead to divergences arising from the kinematical region where one or both bottom quarks are collinear to the incoming partons. The bottom quark mass regulates these divergences, but they still leave traces in terms of logarithms of the form $\ln(m_b^2/m_H^2)$. Such logarithms lead to large perturbative coefficients, so ideally one would like to resum them. This can be achieved by considering this process in the five-flavour scheme (5FS) [18, 19], i.e. by introducing bottom quark PDFs (parton density functions).¹ Now that the b quarks can appear in the initial state, the leading order process is changed to that in Fig. 1(b). We note that the scheme choice amounts merely to a re-ordering of the perturbative series. Of course, when truncated at a finite order, results obtained in either scheme will differ, with the difference being formally of higher order in α_s .

However, it was found that the difference between the inclusive cross section in the 4FS and the 5FS differs by roughly a factor of five when evaluated at $\mu_F = \mu_R = M_H$, where μ_F/μ_R is the factorization/renormalization scale. This remains true also at NLO QCD which was calculated for the 5FS in Ref. [21, 22], and for the 4FS in Ref. [23, 24]. It was thus proposed in Refs. [22, 25–27] that when using the five flavour scheme the appropriate scale choice is $m_H/4$.

A weakness of the 5FS is that it neglects the contribution from large- p_T bottom quarks at leading order. This is taken into account only at NNLO and higher (note that the LO set of Feynman diagrams in the 4FS is part of the NNLO set in the 5FS). Indeed, the factorization scale dependence at NNLO is very flat [28] and seems to confirm the “natural” scale choice at lower orders of $\mu_F = M_H/4$.

It has been pointed out a long time ago that it can be advantageous to consider the $H + \text{jet}$ process instead of the fully inclusive production when searching for the Higgs boson [29]. The $gg \rightarrow H + \text{jet}$ cross section is known at LO, including the full top and bottom quark mass dependence, both in the SM [29–31] and in the MSSM [31]. NLO QCD corrections are known in the heavy-top limit [32–34]. It is expected that a very good approximation of the MSSM effects can be obtained by simply replacing the corresponding Wilson coefficient of the effective $gh\bar{g}$ coupling with its MSSM expression [35–37], at least as long as $\tan\beta$ is not too large. Otherwise, bottom loop effects which are not covered in the heavy-top limit will be important. Resummation effects for small and large p_T of the Higgs boson have been treated in Refs. [38–41].

As mentioned already above, for large $\tan\beta$, it is essential to also take bottom quark annihilation into account. It is well known that the corresponding QCD corrections can be

¹In fact, this has been the default for all modern PDF sets; see Ref. [20] though.



Figure 2: Representative diagrams for each of the two leading order channels.

large. In this paper we present them for distributions of the Higgs boson. Since it has been shown for the inclusive cross section that the dominant SUSY effects can be approximated to high accuracy by an effective $b\bar{b}h$ coupling [42], our results are directly applicable to the MSSM by a trivial overall rescaling.

In Ref. [43], a related quantity, namely the Higgs production cross section in association with a single tagged b quark was studied in the 5FS. Tagging a b may be useful for measuring the $b\bar{b}h$ Yukawa coupling y_b , for example. In this paper we consider Higgs production without necessarily tagging a final state b . That is, we consider the $b\bar{b}$ initial state, and its associated sub-channels, as a contribution to the inclusive Higgs+jet cross section. We present the p_T and y distributions, and study the scale dependence of the cross section and its component channels. These results are valid to NLO in QCD perturbation theory. They will be presented in Section 2.

Using the knowledge of the total inclusive cross section at NNLO [28], we can then use our result for NLO H +jet production in order to derive the NNLO cross section with upper cuts on p_T . This will be described in Section 3.

2 Next-to-leading order cross section for $\sigma(b\bar{b} \rightarrow h + \text{jet})$

The generic leading order diagrams to Higgs plus jet production are shown in Fig. 2. At NLO each of these receives virtual corrections, and in addition we must include the real emission contributions, which induce also other initial states. The full list of processes at NLO is: $b\bar{b} \rightarrow gH$ and $bg \rightarrow bH$ at one loop, $b\bar{b} \rightarrow ggH$, $b\bar{b} \rightarrow b\bar{b}H$, $b\bar{b} \rightarrow q\bar{q}H$, $gb \rightarrow gbH$, $bq \rightarrow bqH$, $q\bar{q} \rightarrow b\bar{b}H$, $gg \rightarrow b\bar{b}H$, $bb \rightarrow bbH$ at tree level, where q denotes any of the light quarks u, d, s, c . It is understood that the charge conjugated processes must also be included. Formally, the virtual contributions include diagrams where the Higgs boson is radiated off a closed bottom or top quark loop. The former lead to terms $\sim \alpha_s^2 y_b^2 \cdot m_b^2 / M_H^2$, however, which is neglected throughout our calculation, and in the spirit of Refs. [28, 43], we discard the latter which are proportional to the top Yukawa coupling. They are separately finite and gauge invariant and could simply be added to our results, once the ratio of top and bottom Yukawa coupling is known.

The NLO calculation of a process as the one considered here is by now standard. We apply the dipole subtraction method [44] in order to cancel the infra-red poles between the virtual and the real radiation contributions in the $b\bar{b}$ and bg processes. Introducing the α -parameter for restricting the dipole phase space [45, 46] not only improves the numerical integration, but also serves as a welcome check through the requirement of α -independence of the final result. Furthermore, our result for the virtual corrections agrees with the result of Ref. [43]. The leading logarithmic behaviour at small p_T can be checked numerically

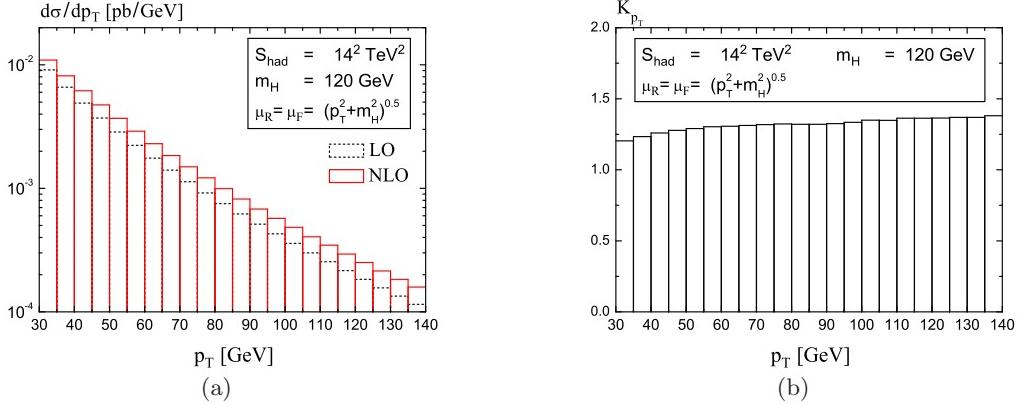


Figure 3: (a) Higgs transverse momentum distribution at LO (dashed) and NLO (solid); (b) corresponding K-factor.

against the resummed expression of Ref. [47]. The most important check, however, is the numerical comparison to a fully analytic evaluation of the p_T distribution to be published elsewhere [48].

In order to avoid the infra-red divergence at low Higgs transverse momenta p_T , we cut contributions from $p_T < 30 \text{ GeV}$ in this section. For our numerical analysis we use the following set of input parameters. The PDFs are taken from the MSTW2008 set [49], and the QCD coupling is accordingly set to $\alpha_s(M_Z) = 0.13939$ at LO, and $\alpha_s(M_Z) = 0.12018$ at NLO. The $b\bar{b}h$ coupling, for which we assume the SM expression m_b/v ($v = 246.22 \text{ GeV}$), is evaluated with the running bottom quark mass $m_b(\mu_R)$ defined in the $\overline{\text{MS}}$ scheme, with an input value $m_b(m_b) = 4.2 \text{ GeV}$ [50]. Our default value for the Higgs mass is $M_H = 120 \text{ GeV}$.

In Fig. 3 and Fig. 4 we show the transverse momentum and rapidity distributions of the Higgs boson, both at LO and NLO, and the corresponding K -factors

$$K_{p_T} \equiv \frac{(d\sigma/dp_T)_{\text{NLO}}}{(d\sigma/dp_T)_{\text{LO}}}, \quad K_y \equiv \frac{(d\sigma/dy)_{\text{NLO}}}{(d\sigma/dy)_{\text{LO}}}, \quad (1)$$

where

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \quad (2)$$

and E and p_z are the energy and longitudinal component of the Higgs boson in the lab frame. The choice of the renormalization and factorization scales is given in the plots. We remark that the numerator/denominator in Eq. (1) is evaluated with NLO/LO PDFs and couplings. Both for the p_T and the y distribution, the dependence of the K factors on p_T and y is very similar to what is observed for the gluon fusion channel [32–34]: K_{p_T} is rather flat over the considered p_T interval, while K_y drops mildly towards larger values of the rapidity. The absolute size of the corrections is significantly smaller than in the gluon fusion case though. Note that the K factors for the distributions cannot be immediately deduced from the one for the inclusive cross section for $b\bar{b} \rightarrow H + X$ due to the strong μ_F dependence at LO, cf. Ref. [28] and Fig. 6 (b).

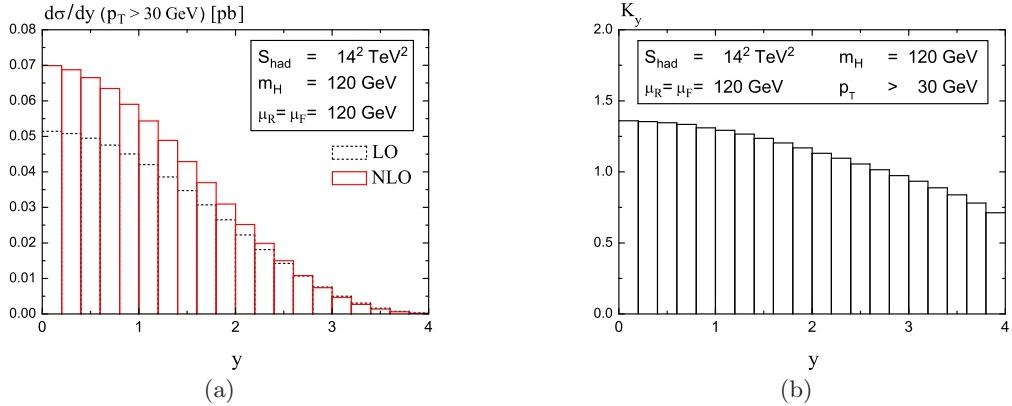


Figure 4: (a) Higgs rapidity distribution at LO (dashed) and NLO (solid); (b) corresponding K-factor. Since the distribution is symmetric around $y = 0$, only positive values of y are shown.

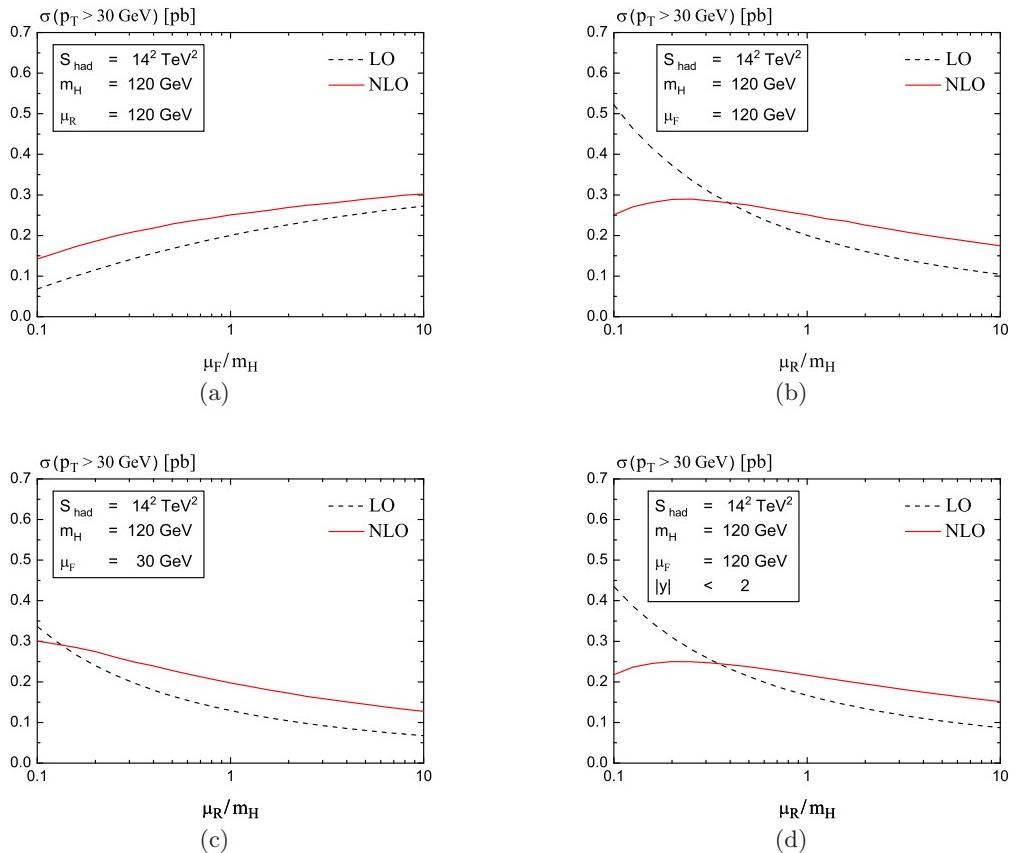


Figure 5: Scale dependence of the integrated cross section for $p_T > 30 \text{ GeV}$ at $M_H = 120 \text{ GeV}$. (a) $\mu_R = M_H$ fixed, μ_F varies — (b) $\mu_F = M_H$ fixed, μ_R varies — (c) $\mu_F = M_H/4$ fixed, μ_R varies — (d) same as (b), but with cut on $|y| < 2$.

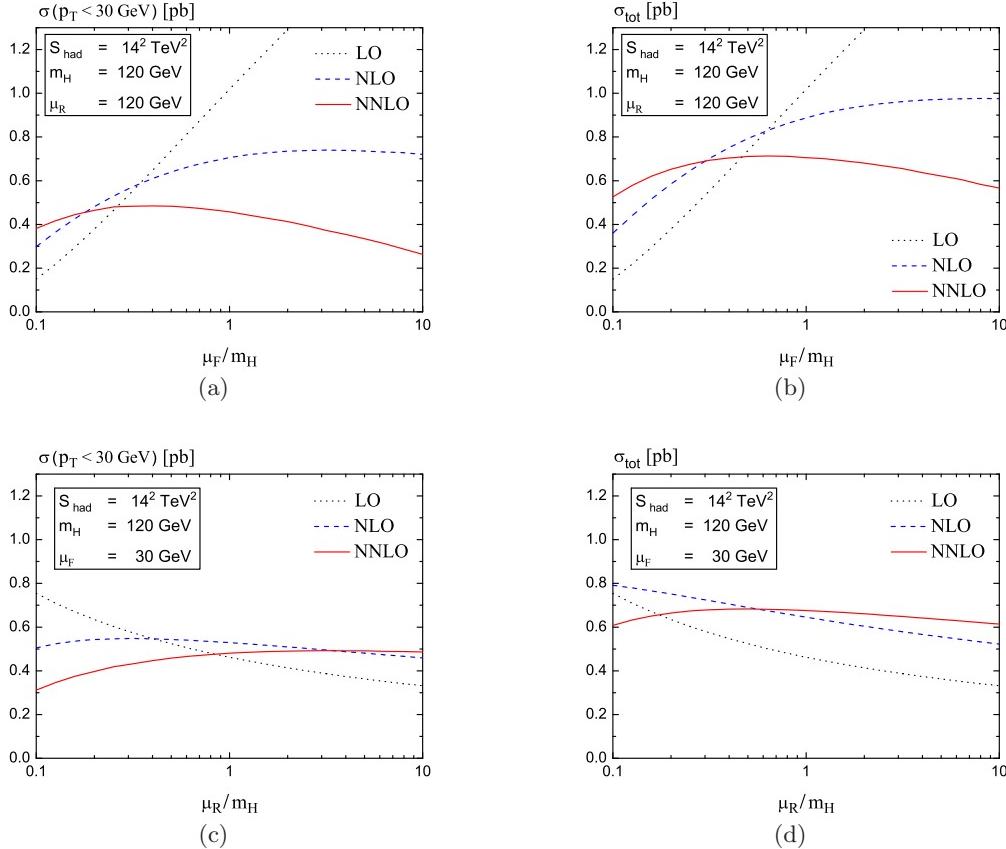


Figure 6: Scale dependence of the integrated cross section for $p_T < 30 \text{ GeV}$ at $M_H = 120 \text{ GeV}$. (a) $\mu_R = M_H$ fixed, μ_F varies — (b) $\mu_F = M_H/4$ fixed, μ_R varies.

In Fig. 5 we show the integrated cross section for $h + \text{jet}$ production with a minimum p_T of the Higgs of $p_{T,\text{cut}} = 30 \text{ GeV}$:

$$\sigma(p_T > p_{T,\text{cut}}) = \int_{p_T > p_{T,\text{cut}}} dp_T \frac{d\sigma}{dp_T} \quad (3)$$

at $M_H = 120 \text{ GeV}$ as a function of (a) the factorization and (b,c) the renormalization scale. The factorization scale dependence is already quite small at LO and improves slightly at NLO. The renormalization scale dependence we show for (b) $\mu_F = M_H$ and (c) $\mu_F = M_H/4$, respectively. For both choices, the NLO corrections improve the scale dependence significantly, although $\mu_F = M_H$ seems to lead to a more natural behaviour of the LO and the NLO curves. This behaviour does not change much if we restrict the Higgs rapidity to $|y| < 2$, as shown in Fig. 5(d).

3 NNLO cross section with p_T cut

The knowledge of the total inclusive cross section σ_{tot} at NNLO [28] allows us to use the results of the current paper to evaluate the inclusive cross section when applying a finite

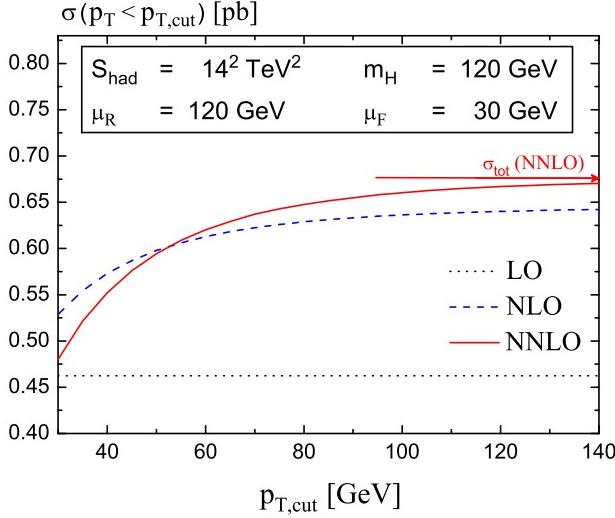


Figure 7: Inclusive cross section with an upper cut $p_{T,\text{cut}}$ on the Higgs transverse momentum, see Eq. (4), as a function of $p_{T,\text{cut}}$. The dotted, dashed, and solid curves show the LO, NLO, and the NNLO result. The arrow on the right indicates the value of the NNLO result without cuts, σ_{tot} .

p_T cut:

$$\sigma(p_T < p_{T,\text{cut}}) = \int_{p_T < p_{T,\text{cut}}} dp_T \frac{d\sigma}{dp_T} = \sigma_{\text{tot}} - \int_{p_T > p_{T,\text{cut}}} dp_T \frac{d\sigma}{dp_T}. \quad (4)$$

Of course, $p_{T,\text{cut}}$ must not be too small in order not to be sensitive to the region where large $\ln(p_T/M_H)$ terms spoil perturbative convergence. Furthermore, if $\sigma(p_T < p_{T,\text{cut}})$ is to be evaluated with NNLO accuracy, we have to evaluate both terms on the right side of Eq. (4) with NNLO PDFs.

Fig. 6 (a) shows $\sigma(p_T < p_{T,\text{cut}})$ for $p_{T,\text{cut}} = 30$ GeV as a function of the factorization scale μ_F , for $\mu_R = M_H$, varied over the rather large interval $\mu_F \in [0.1, 10]M_H$. The observations are very similar to the fully inclusive cross section obtained without cuts [28] which is shown for comparison in Fig. 6 (b): Perturbation theory prefers a scale significantly below M_H . With this choice, the renormalization scale dependence is very weak around $\mu_R = M_H$ already at NLO, as shown in Fig. 6 (c) and Fig. 6 (d).

Finally, Fig. 7 shows the NNLO cross section $\sigma(p_T < p_{T,\text{cut}})$ as a function of $p_{T,\text{cut}}$. Since the LO only contributes at $p_T = 0$, it is independent on $p_{T,\text{cut}}$. The NNLO corrections are negative with respect to NLO at small $p_{T,\text{cut}}$ and change sign at around $p_{T,\text{cut}} \approx 50$ GeV.

4 Conclusions and Outlook

In this paper we have presented a first study of higher order differential distributions for Higgs production in bottom quark annihilation. The five-flavour scheme was used to calculate NLO p_T and y distributions of the Higgs boson. Combination with the inclusive

NNLO total cross section allowed us to derive the inclusive cross section with upper cuts on the Higgs transverse momentum at NNLO.

Concerning the choice of the factorization scale, we find that it strongly depends on the observable under consideration: the value $\mu_F = M_H/4$ [22, 26, 28] seems to be favoured in particular when the region $p_T = 0$ is involved, but is less motivated otherwise.

Although we expect that the results will be phenomenologically relevant on their own, our approach should be useful also for an extension to a fully differential NNLO Monte Carlo program for the process $b\bar{b} \rightarrow H + X$ along the lines of Ref. [51].

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References

- [1] G.S. Guralnik, C.R. Hagen, T.W.B. Kibble, *Global conservation laws and massless particles*, *Phys. Rev. Lett.* **13** (1964) 585.
- [2] F. Englert and R. Brout, *Broken symmetry and the mass of gauge vector mesons*, *Phys. Rev. Lett.* **13** (1964) 321.
- [3] P.W. Higgs, *Broken symmetries and the masses of gauge bosons*, *Phys. Rev. Lett.* **13** (1964) 508.
- [4] S.L. Glashow, *Partial symmetries of weak interactions*, *Nucl. Phys.* **B 22** (1961) 579.
- [5] S. Weinberg, *A Model Of Leptons*, *Phys. Rev. Lett.* **19** (1967) 1264.
- [6] A. Salam, in: *Elementary Particle Theory: Relativistic Groups and Analyticity*, ed. N. Svartholm, Almqvist and Wiksell, Stockholm (1968) p.367.
- [7] H.P. Nilles, *Supersymmetry, Supergravity And Particle Physics*, *Phys. Reports* **110** (1984) 1.
- [8] The LEP Working Group for Higgs boson searches, *Search for the standard model Higgs boson at LEP*, *Phys. Lett.* **B 565** (2003) 61.
- [9] T. Aaltonen *et al.* [CDF and D0 Collaborations], *Combination of Tevatron searches for the standard model Higgs boson in the W+W- decay mode*, *Phys. Rev. Lett.* **104** (2010) 061802.
- [10] LEP Electroweak Working Group, <http://lepewwg.web.cern.ch/LEPEWWG/>
- [11] The LEP Working Group for Higgs Boson Searches, *Search for neutral MSSM Higgs bosons at LEP*, *Eur. Phys. J. C* **47** (2006) 547.
- [12] Tevatron New Phenomena & Higgs Working Group, *Combined CDF and D0 upper limits on MSSM Higgs boson production in $\tau\tau$ final states with up to 2.2 fb^{-1}* , arXiv:1003.3363.

- [13] A. Djouadi, *The anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model*, *Phys. Reports* **457** (2008) 1.
- [14] A. Djouadi, *The anatomy of electro-weak symmetry breaking. II: The Higgs bosons in the minimal supersymmetric model*, *Phys. Reports* **459** (2008) 1.
- [15] A. Belyaev, A. Blum, R.S. Chivukula, E.H. Simmons, *The meaning of Higgs: $\tau^+\tau^-$ and $\gamma\gamma$ at the Tevatron and the LHC*, *Phys. Rev. D* **72** (2005) 055022.
- [16] A. Belyaev, J. Pumplin, W.K. Tung, C.P. Yuan, *Uncertainties of the inclusive Higgs production cross section at the Tevatron and the LHC*, *JHEP* **0601** (2006) 069.
- [17] O. Brein, *Electroweak and Bottom Quark Contributions to Higgs Boson plus Jet Production*, *Phys. Rev. D* **81** (2010) 093006.
- [18] R.M. Barnett, H.E. Haber, D.E. Soper, *Ultraheavy particle production from heavy partons at hadron colliders*, *Nucl. Phys. B* **306** (1988) 697.
- [19] D.A. Dicus and S. Willenbrock, *Higgs boson production from heavy quark fusion*, *Phys. Rev. D* **39** (1989) 751.
- [20] A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, *Heavy-quark mass dependence in global PDF analyses and 3- and 4-flavour parton distributions*, [arXiv:1007.2624](https://arxiv.org/abs/1007.2624).
- [21] D. Dicus, T. Stelzer, Z. Sullivan, S. Willenbrock, *Higgs boson production in association with bottom quarks at next-to-leading order*, *Phys. Rev. D* **59** (1999) 094016.
- [22] F. Maltoni, Z. Sullivan, S. Willenbrock, *Higgs-boson production via bottom-quark fusion*, *Phys. Rev. D* **67** (2003) 093005.
- [23] S. Dittmaier, M. Krämer, M. Spira, *Higgs radiation off bottom quarks at the Tevatron and the LHC*, *Phys. Rev. D* **70** (2004) 074010.
- [24] S. Dawson, C.B. Jackson, L. Reina, D. Wackerlo, *Exclusive Higgs boson production with bottom quarks at hadron colliders*, *Phys. Rev. D* **69** (2004) 074027.
- [25] D. Rainwater, M. Spira, D. Zeppenfeld, *Higgs boson production at hadron colliders: Signal and background processes*, [hep-ph/0203187](https://arxiv.org/abs/hep-ph/0203187).
- [26] T. Plehn, *Charged Higgs boson production in bottom-gluon fusion*, *Phys. Rev. D* **67** (2003) 014018.
- [27] E. Boos and T. Plehn, *Higgs-boson production induced by bottom quarks*, *Phys. Rev. D* **69** (2004) 094005.
- [28] R.V. Harlander and W.B. Kilgore, *Higgs boson production in bottom quark fusion at next-to-next-to-leading order*, *Phys. Rev. D* **68** (2003) 013001.
- [29] R. K. Ellis, I. Hinchliffe, M. Soldate, J. J. van der Bij, *Higgs Decay To $\tau^+\tau^-$: A Possible Signature Of Intermediate Mass Higgs Bosons At The SSC*, *Nucl. Phys. B* **297** (1988) 221.
- [30] B. Field, S. Dawson, J. Smith, *Scalar and pseudoscalar Higgs boson plus one jet production at the LHC and Tevatron*, *Phys. Rev. D* **69** (2004) 074013.

- [31] O. Brein and W. Hollik, *MSSM Higgs bosons associated with high- p_T jets at hadron colliders*, *Phys. Rev.* **D 68** (2003) 095006.
- [32] D. de Florian, M. Grazzini, Z. Kunszt, *Higgs production with large transverse momentum in hadronic collisions at next-to-leading order*, *Phys. Rev. Lett.* **82** (1999) 5209.
- [33] V. Ravindran, J. Smith, W.L. van Neerven, *Next-to-leading order QCD corrections to differential distributions of Higgs boson production in hadron hadron collisions*, *Nucl. Phys.* **B 634** (2002) 247.
- [34] C.J. Gossler and C.R. Schmidt, *Next-to-leading corrections to the Higgs boson transverse momentum spectrum in gluon fusion*, *JHEP* **0212** (2002) 016.
- [35] R.V. Harlander and M. Steinhauser, *Hadronic Higgs Production and Decay in Supersymmetry at Next-to-Leading Order*, *Phys. Lett.* **B 574** (2003) 258-268.
- [36] R.V. Harlander and M. Steinhauser, *Supersymmetric Higgs production in gluon fusion at next-to-leading order*, *JHEP* **0409** (2004) 066.
- [37] G. Degrassi and P. Slavich, *On the NLO QCD corrections to Higgs production and decay in the MSSM*, *Nucl. Phys.* **B 805** (2008) 267.
- [38] A. Kulesza, G. Sterman, W. Vogelsang, *Joint resummation for Higgs production*, *Phys. Rev.* **D 69** (2004) 014012.
- [39] G. Bozzi, S. Catani, D. de Florian, M. Grazzini, *The q_T spectrum of the Higgs boson at the LHC in QCD perturbation theory*, *Phys. Lett.* **B 564** (2003) 65.
- [40] G. Bozzi, S. Catani, D. de Florian, M. Grazzini, *Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC*, *Nucl. Phys.* **B 737** (2006) 73.
- [41] D. de Florian, A. Kulesza, W. Vogelsang, *Threshold resummation for high-transverse-momentum Higgs production at the LHC*, *JHEP* **0602** (2006) 047.
- [42] S. Dittmaier, M. Krämer, A. Mück, T. Schlüter, *MSSM Higgs-boson production in bottom-quark fusion: Electroweak radiative corrections*, *JHEP* **0703** (2007) 114.
- [43] J. Campbell, R.K. Ellis, F. Maltoni, S. Willenbrock, *Higgs boson production in association with a single bottom quark*, *Phys. Rev.* **D 67** (2003) 095002.
- [44] S. Catani and M.H. Seymour, *A general algorithm for calculating jet cross sections in NLO QCD*, *Nucl. Phys.* **B 485** (1997) 291; (E) *ibid.* **510** (1998) 503.
- [45] Z. Nagy, *Next-to-leading order calculation of three jet observables in hadron hadron collision*, *Phys. Rev.* **D 68** (2003) 094002.
- [46] Z. Nagy and Z. Trócsányi, *Next-to-leading order calculation of four-jet observables in electron positron annihilation*, *Phys. Rev.* **D 59** (1999) 014020; (E) *ibid.* **62** (2000) 099902.
- [47] A. Belyaev, P.M. Nadolsky, C.P. Yuan, *Transverse momentum resummation for Higgs boson produced via $b\bar{b}$ fusion at hadron colliders*, *JHEP* **0604** (2006) 004.

- [48] K.J. Ozeren, *in preparation.*
- [49] A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, *Parton distributions for the LHC*, *Eur. Phys. J. C* **63** (2009) 189.
- [50] C. Amsler *et al.* [Particle Data Group], *Review of particle physics*, *Phys. Lett. B* **667** (2008) 1.
- [51] S. Catani and M. Grazzini, *An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC*, *Phys. Rev. Lett.* **98** (2007) 222002.